

1. Prove or disprove: if a , b , and c are positive integers such that $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. (Hint: use the GCD theorem.)

2. Suppose $ax + by = 13$. Which of the following possibilities can occur (if any)? In each case, either give an example or a proof that it cannot occur.

- (a) $\gcd(a, b) = 1, \gcd(x, y) = 1$
- (b) $\gcd(a, b) = 1, \gcd(x, y) = 13$
- (c) $\gcd(a, b) = 13, \gcd(x, y) = 1$
- (d) $\gcd(a, b) = 13, \gcd(x, y) = 13$