Quiz 10 - Math 54
November 10, 2010

Name Solution

Consider the system of differential equations $\begin{aligned} x_1' &= 4x_1 - 2x_2 \\ x_2' &= 6x_1 - 3x_2 \end{aligned}$

1)[6pts] Find the general solution to the system.

The associated matrix is $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$. We find the eigenvalues and eigenvectors: $\det(A - \lambda I) = \det \begin{bmatrix} 4-\lambda & -2 \\ 6 & -3-\lambda \end{bmatrix} = \lambda^2 - \lambda = \lambda(\lambda - 1)$, so the eigenvalues are 0 and 1. $A - OI = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$, so $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a 0-eigenvector. $A - 1I = \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix} \sim \begin{bmatrix} 3 & -2 \\ 0 & 0 \end{bmatrix}$, so $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a 1-eigenvector. So the general solution is

$$\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^t \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

2)[4pts] Find two **specific** solutions which are linearly independent and prove they are linearly independent.

Taking $c_1 = 1$ and $c_2 = 0$, we get the solution $\mathbf{x}(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Taking $c_1 = 0$ and $c_2 = 1$, we get $\mathbf{x}(t) = \begin{bmatrix} 2e^t \\ 3e^t \end{bmatrix}$. To show these are linearly independent, we compute the Wronskian

$$\det \begin{bmatrix} 1 & 2e^t \\ 2 & 3e^t \end{bmatrix} = -e^t$$

Since the Wronskian is not zero, the two solutions are linearly independent. (It's also enough to simply check that the Wronskian is not zero for one value of t.)