Quiz 9 - Math 54
November 3, 2010

Name______________________________

1)[5pts] Find the general solution to $y'' - y = 5e^t$.

The auxiliary equation is $r^2 - 1 = (r+1)(r-1) = 0$, so the solution to the homogeneous equation is $c_1e^t + c_2e^{-t}$. Since e^t is a solution to the homogeneous equation, we try $y = Ate^t$. Then $y' = Ate^t + Ae^t$ and $y'' = Ate^t + 2Ae^t$, so we have

$$y'' - y = 2Ae^t = 5e^t$$

so $A = 5/2$. So the general solution is $c_1e^t + c_2e^{-t} + \frac{5}{2}te^t$.

2)[5pts] The functions 1 and t^2 are solutions to the differential equation $y'' - \frac{1}{t}y' = 0$. Find the general solution to $y'' - \frac{1}{t}y' = 2\sqrt{t}$.

We apply variation of parameters, guessing a solution of the form $v_1 + t^2v_2$. We get

$$1 \cdot v_1' + t^2v_2' = 0$$
$$0 \cdot v_1' + 2tv_2' = 2t^{1/2}$$

So $v_2' = t^{-1/2}$ and $v_1' = -t^{3/2}$, so take $v_2 = 2t^{1/2}$ and $v_1 = -\frac{2}{5}t^{5/2}$. Thus $-\frac{2}{5}t^{5/2} + 2t^{5/2} = \frac{8}{5}t^{5/2}$ is a particular solution, so $\frac{8}{5}t^{5/2} + c_1 + c_2t^2$ is the general solution.