

Quiz 6 - Math 54  
October 6, 2010

Name \_\_\_\_\_

$$A = \begin{bmatrix} -1 & -1 & 2 \\ 0 & -1 & 2 \\ 4 & 1 & -2 \end{bmatrix}.$$

1)[5pts] Find the eigenvalues of  $A$ . (They're all integers.)

$$\det(A - \lambda I) = \det \begin{bmatrix} -1 - \lambda & -1 & 2 \\ 0 & -1 - \lambda & 2 \\ 4 & 1 & -2 - \lambda \end{bmatrix} = (-1 - \lambda)(2 + 3\lambda + \lambda^2 - 2) + 4(-2 + 2 + 2\lambda) =$$

$$5\lambda - 4\lambda^2 - \lambda^3 = -\lambda(\lambda + 5)(\lambda - 1). \text{ So the eigenvalues of } A \text{ are } \boxed{0, 1, \text{ and } -5}.$$

2)[5pts] Find an invertible matrix  $P$  so that  $P^{-1}AP$  is diagonal. What is  $P^{-1}AP$ ? (Hint: once you find  $P$ , you can find  $P^{-1}AP$  without any additional computation ... in particular, you don't actually need to find  $P^{-1}$ .)

$P^{-1}AP$  will be diagonal if the columns of  $P$  are a basis of eigenvectors of  $A$ . So we find an eigenvector for each eigenvalue:

$$\begin{bmatrix} -1 & -1 & 2 \\ 0 & -1 & 2 \\ 4 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -2 \\ 4 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}, \text{ so } \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \text{ is a 0-eigenvector.}$$

$$\begin{bmatrix} -2 & -1 & 2 \\ 0 & -2 & 2 \\ 4 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} -2 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \text{ so } \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \text{ is a 1-eigenvector.}$$

$$\begin{bmatrix} 4 & -1 & 2 \\ 0 & 4 & 2 \\ 4 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 4 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & 0 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \text{ so } \begin{bmatrix} -5 \\ -4 \\ 8 \end{bmatrix} \text{ is a } (-5)\text{-eigenvector.}$$

Taking  $P = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 2 & -4 \\ 1 & 2 & 8 \end{bmatrix}$ , we have  $P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix}.$