Quiz 4 - Math 54
September 22, 2010

Name______________________

A is the matrix $\begin{bmatrix} 1 & 0 & -2 & -1 & 2 \\ 1 & 1 & -1 & -2 & 1 \\ 1 & 1 & -1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

1)[5pts] Find a basis for $Col(A)$. (Hint: row operations do not change the linear relations among the columns of a matrix.)

Row reducing, we have $\begin{bmatrix} 1 & 0 & -2 & -1 & 2 \\ 1 & 1 & -1 & -2 & 1 \\ 1 & 1 & -1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 & 2 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 & 2 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. We see that the third and fourth columns can be expressed as linear cominations of the first and second columns, and that the first, second, and fifth columns are linearly independent. So the three vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ form a basis for $Col(A)$.

2)[5pts] Find a basis for $Nul(A)$. (Hint: row operations do not change the linear relations among the columns of a matrix.)

The null space of A is the set of vectors $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$ so that $A\mathbf{x} = \mathbf{0}$. From the row reduction, we see that x_3 and x_4 are free variables, while x_1, x_2, and x_5 are bound. Specifically, $x_1 = 2x_3 + x_4$, $x_2 = -x_3 + x_4$, and $x_5 = 0$, so any element of the null space is of the form $x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$. Thus, $\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ form a basis for $Nul(A)$.