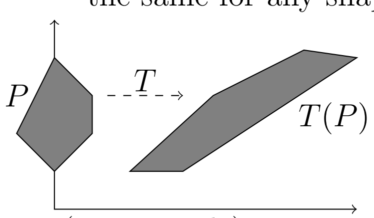
Quiz 3 - Math 54
September 15, 2010

Name________________________________

T is the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by $T(x, y) = (x + 2y, x + y)$.
1)[5pts] Find the inverse of T and compute $T^{-1}(3, 4)$. (Hint: express T as a matrix.)

$T(1, 0) = (1, 1)$ and $T(0, 1) = (2, 1)$, so the standard matrix of T is $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. The inverse of this matrix is $\frac{1}{1-2}\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$, so the inverse of T is given by $T^{-1}(x, y) = (-x + 2y, x - y)$. Specifically $T^{-1}(3, 4) = (5, -1)$.

2)[5pts] The pentagon P with vertices $(0, 1)$, $(1, 2)$, $(1, 3)$, $(0, 4)$, and $(-1, 2)$ has area $\frac{7}{2}$. Find the area of the image $T(P)$. (Hint: the exact shape doesn't matter ... the answer would be the same for any shape of area $\frac{7}{2}$.)



(not to scale)

A linear transformation scales area by the absolute value of its determinant. Since we computed that the matrix of T is $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, the determinant is -1, so T scales area by a factor of 1. Therefore, the area of $T(P)$ is $\frac{7}{2}$.