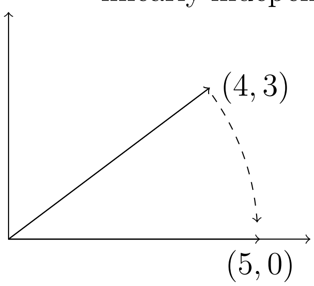
Quiz 2 - Math 54
September 8, 2010

Name________________________________

1)[5pts] Let T be the rotation of $\mathbb{R}^2$ around the origin which takes $\begin{bmatrix}4\\3\end{bmatrix}$ to $\begin{bmatrix}5\\0\end{bmatrix}$. Find the standard matrix of T. (Hint: one way to do this is to figure out what T does to two specific linearly independent vectors, then express $\mathbf{e}_1$ and $\mathbf{e}_2$ as linear combinations of those vectors.)



The rotation takes $5\mathbf{e}_2 = \begin{bmatrix}0\\5\end{bmatrix}$ to $\begin{bmatrix}3\\4\end{bmatrix}$ and $5\mathbf{e}_1 = \begin{bmatrix}5\\0\end{bmatrix}$ to $\begin{bmatrix}4\\-3\end{bmatrix}$. By linearity, $T(\mathbf{e}_1) = \begin{bmatrix}4/5\\-3/5\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}3/5\\4/5\end{bmatrix}$, so the standard matrix of T is $\begin{bmatrix}4/5 & 3/5\\-3/5 & 4/5\end{bmatrix}$.

2)[5pts] Suppose $AB = \begin{bmatrix}5 & 4\\-2 & 3\end{bmatrix}$ and $B = \begin{bmatrix}7 & 3\\2 & 1\end{bmatrix}$. Find A.

Either by row reducing $[B\ I]$ or by using the formula of Theorem 4, we compute $B^{-1} = \frac{1}{7\cdot 1 - 3\cdot 2}\begin{bmatrix}1 & -3\\-2 & 7\end{bmatrix} = \begin{bmatrix}1 & -3\\-2 & 7\end{bmatrix}$. Then $A = AB \cdot B^{-1} = \begin{bmatrix}5 & 4\\-2 & 3\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 7\end{bmatrix} = \begin{bmatrix}-3 & 13\\-8 & 27\end{bmatrix}$.