A. Suppose A is an $n \times n$ matrix. Show that A is symmetric (i.e. $A = A^T$) if and only if $A\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot A\mathbf{y}$ for all vectors $\mathbf{x}$ and $\mathbf{y}$. (Hint: Find vectors $\mathbf{x}$ and $\mathbf{y}$ so that $A\mathbf{x} \cdot \mathbf{y}$ is the (i, j)-th entry of A)

B. Let $p(t) = 1$ and $q(t) = t$. Find $\mathrm{proj}_p\, q$ under the usual inner product on $C[0, 1]$ ($\langle f, g\rangle = \int_0^1 f(t)g(t)\, dt$).

C. Determine $\|1 - 3t^2\|$ under the usual inner product on $C[0, 1]$ ($\langle f, g\rangle = \int_0^1 f(t)g(t)\, dt$).

D. For a pair of $n \times n$ matrices A and B, define $\langle A, B\rangle = \det(AB)$. Is this an inner product? For a pair of functions f and g, define $\langle f, g\rangle = f(0)g(0) + f(1)g(1) + f(2)g(2)$. Is this an inner product?

E. Let $p(t) = 1$ and $q(t) = t$. Find $\mathrm{proj}_p\, q$ under the inner product $\langle f, g\rangle = f(0)g(0)+f(1)g(1)+f(2)g(2)$.

F. Suppose $A = A^T$, $A\mathbf{v} = \lambda\mathbf{v}$, and $A\mathbf{w} = \mu\mathbf{w}$. Show that $\lambda(\mathbf{v} \cdot \mathbf{w}) = \mu(\mathbf{v} \cdot \mathbf{w})$.