

A. $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for \mathbb{R}^3 . Let $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Find $[\mathbf{v}]_{\mathcal{B}}$.

B. Suppose $\mathcal{B} = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is a basis for \mathbb{R}^n . Write a matrix equation relating a vector \mathbf{x} to $[\mathbf{x}]_{\mathcal{B}}$.
(Hint: Say $n = 3$ and $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$... what is \mathbf{x} ? Can you express that as a matrix equation?)

C. Use coordinates to determine if $\{1 - 3t + 5t^2, -3 + 5t - 7t^2, -4 + 5t - 6t^2, 1 - t^2\}$ spans \mathbb{P}_2 .

D. Find the dimension of the span of the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$. (Recall: the dimension of a vector space is the number of elements in a basis.)

E. What is the dimension of the vector space of 2×2 matrices? Find a basis consisting of invertible matrices. (Recall: the dimension of a vector space is the number of elements in a basis.)

F. $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for \mathbb{R}^3 . Let A be a matrix so that $A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$,
 $A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$. Find A .