Fake Quiz - Math 54
November 24, 2010

Name ____Solution____

1) Consider the differential equation $\dfrac{\partial^2 u}{\partial x^2} + \left(\dfrac{\partial u}{\partial x}\right)^2 \dfrac{\partial u}{\partial t} = 0$. If you're looking for solutions of the form $u(x,t) = X(x)T(t)$, what differential equations would $X(x)$ and $T(t)$ have to satisfy? (Note: you don't have to solve those, just find them.)

Plugging $X(x)T(t)$ in, we get the equation $X''(x)T(t) + X'(x)^2X(x)T'(t)T(t)^2 = 0$. Separating the variables, we get that there is a constant K so that $\frac{-X''(x)}{X'(x)^2X(x)} = K$ (or $X''(x) + kX'(x)^2X(x) = 0$) and $T'(t)T(t) = K$.

2) Compute the Fourier series of $|\sin x|$ on the interval $[-\pi, \pi]$. (You may find the identities

$$\sin\theta\sin\phi = \frac{1}{2}\big(\cos(\theta-\phi) + \cos(\theta+\phi)\big)$$
$$\sin\theta\cos\phi = \frac{1}{2}\big(\sin(\theta-\phi) + \sin(\theta+\phi)\big)$$

useful.)

The function is even, so we compute $a_0 = \frac{2}{\pi}\int_0^\pi \sin(x)dx = \frac{2}{\pi}\cdot 2 = \frac{4}{\pi}$, $a_1 = \frac{2}{\pi}\int_0^\pi \sin(x)\cos(x)dx = \frac{1}{\pi}\int_0^\pi \sin(2x)dx = 0$, and for $n > 1$,

$$\begin{aligned}
a_n &= \frac{2}{\pi}\int_0^\pi \sin(x)\cos(nx)dx = \frac{1}{\pi}\int_0^\pi (\sin((1+n)x) + \sin((1-n)x)) \\
&= \frac{1}{\pi}\left[\frac{-1}{1+n}\cos((1+n)x) + \frac{-1}{1-n}\cos((1-n)x)\right]_0^\pi \\
&= \frac{1}{\pi}\left(\frac{1}{1+n} + \frac{1}{1-n}\right)((-1)^{n+1}+1) = \frac{1}{\pi}\cdot\frac{2}{1-n^2}((-1)^{n+1}+1).
\end{aligned}$$

So the Fourier series is

$$\frac{2}{\pi} + \frac{4\cos(3x)}{\pi(1-3^2)} + \frac{4\cos(5x)}{\pi(1-5^2)} + \cdots = \frac{2}{\pi} + \sum_{k=1}^{\infty}\frac{4\cos((2k+1)x)}{\pi(1-(2k+1)^2)}$$