Quiz 8 - Math 53
October 30, 2008

Name______________________________

1)[4pts] Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{x^2+y^2} 1\, dz\, dy\, dx$. (Hint: convert to cylindrical coordinates and write the integral in the order $dz\, dr\, d\theta$)

The integral is

$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{1} \int_{z=0}^{r^2} 1 \cdot r\, dz\, dr\, d\theta = \int_{\theta=0}^{\pi/2} \int_{r=0}^{1} r^3\, dr\, d\theta = \int_{\theta=0}^{\pi/2} \frac{1}{4}\, d\theta = \frac{\pi}{8}.$$

2)[5pts] Let E be the part of the unit ball where x, y and z are positive, i.e. E is one eighth of the ball. Assuming E has density 1, find the center of mass of E.
(Hint 1: by symmetry, you know that $\bar{x} = \bar{y} = \bar{z}$, so it is enough to find $\bar{z}$.)
(Hint 2: use spherical coordinates.)
(Hint 3: the mass is equal to the volume of E, which you can compute without doing any integrals.)

The mass of E is equal to it's volume, which is one eighth of the volume of the unit sphere, so the mass of E is $\frac{1}{8} \cdot \frac{4}{3} \cdot \pi \cdot 1^3 = \pi/6$.

$$\bar{z} = \frac{6}{\pi} \int_{\phi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} \int_{\rho=0}^{1} \overbrace{\rho\cos\phi}^{z} \cdot \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \frac{6}{\pi} \int_{\phi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} \frac{1}{4} \overbrace{\cos\phi \sin\phi}^{\frac{1}{2}\sin(2\phi)}\, d\theta\, d\phi$$

$$= \frac{6}{\pi} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \int_{\phi=0}^{\pi/2} \sin(2\phi)\, d\phi = \frac{3}{8}\Big[-\frac{1}{2}\cos(2\phi)\Big]_{\phi=0}^{\pi/2} = \frac{3}{8}$$

So the center of mass is $\left(\frac{3}{8}, \frac{3}{8}, \frac{3}{8}\right)$.