Quiz 7 - Math 53
October 24, 2008

Name______________________________

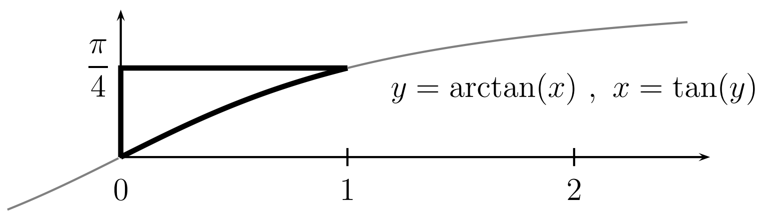
1)[3pts] Compute the integral $\int_0^\infty \int_2^6 e^{-y^2 x}\, dy\, dx$.

The function is everywhere continuous, so we may switch the limits:

$$\int_2^6 \int_0^\infty e^{-y^2 x}\, dx\, dy = \int_2^6 \left[\frac{-1}{y^2} e^{-y^2 x}\right]_{x=0}^{\infty} dy = \int_2^6 \frac{1}{y^2}\, dy$$
$$= \left[\frac{-1}{y}\right]_{y=2}^{6} = \frac{-1}{6} + \frac{1}{2} = \frac{1}{3}$$

2)[3pts] Compute the integral $\int_0^1 \int_{\arctan(x)}^{\pi/4} \sec(y)\, dy\, dx$. (Hints: 1: Switch the order of integration. 2: When the time comes, recall that $\frac{d}{dx}(\sec(x)) = \tan(x)\sec(x)$)

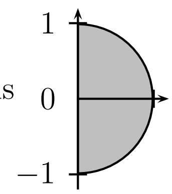
The domain is shown here:



We rewrite the integral as

$$\int_{y=0}^{\pi/4} \int_{x=0}^{\tan(y)} \sec(y)\, dx\, dy = \int_0^{\pi/4} \tan(y) \cdot \sec(y)\, dy$$
$$= \sec(y)\Big|_0^{\pi/4} = \sqrt{2} - 1$$

3)[3pts] Compute $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.

Domain is 

so when we convert to polar coordinates, we have

$$\int_{-\pi/2}^{\pi/2} \int_0^1 e^{r^2} r\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} \int_0^1 \frac{1}{2} e^u\, du\, d\theta = \frac{\pi}{2}(e-1)$$