Quiz 5 - Math 53
October 2, 2008

Name________________________

1)[4pts] Prove that $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - y^3}{\sqrt{x^2+y^2+4}-2} = 0$. (Hint: use the conjugation trick and then switch to polar coordinates.)

$$\begin{aligned}
\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{\sqrt{x^2+y^2+4}-2} &= \lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{\sqrt{x^2+y^2+4}-2} \cdot \frac{\sqrt{x^2+y^2+4}+2}{\sqrt{x^2+y^2+4}+2} \\
&= \lim_{(x,y)\to(0,0)} \frac{(x^3-y^3)(\sqrt{x^2+y^2+4}+2)}{x^2+y^2} \\
&= \lim_{r\to 0} \frac{r^3(\cos^3\theta - \sin^3\theta)(\sqrt{r^2+4}+2)}{r^2} = 0
\end{aligned}$$

2)[5pts] The ideal gas law says that $P = \dfrac{xt}{v}$, where P is pressure, x is the number of gas molecules, t is temperature, and v is volume (all measured in appropriate units). Suppose you've measured $x = 10$, $t = 200$, and $v = 1$, with each measurement accurate to within 1%. What should the value of P be? Using linear approximation, estimate the **maximum** possible error in P. (Hint: to estimate maximum possible error, you may have to assume your errors for x, t, v have different signs. For example, take $\Delta v = .01$ or $\Delta v = -.01$, whichever makes ΔP bigger.)

The measurement of P should be $10 \cdot 200/1 = 2000$. We have that

$$\begin{aligned}
\Delta P &\approx \frac{\partial P}{\partial x}\Delta x + \frac{\partial P}{\partial t}\Delta t + \frac{\partial P}{\partial v}\Delta v \\
&= \frac{t}{v}\Delta x + \frac{x}{v}\Delta t - \frac{xt}{v^2}\Delta v \\
&= 200\Delta x + 10\Delta t - 2000\Delta v
\end{aligned}$$

Because the measurements of x, t, and v are accurate to within 1%, we have that $\Delta x = \pm.1$, $\Delta t = \pm 2$, and $\Delta v = \pm.01$. The error ΔP is maximized when we take $\Delta x = .1$, $\Delta t = 2$, and $\Delta v = -.01$. In this case, $\Delta P = 20 + 20 + 20 = 60$. So the expected measurement of P could be off by as much as 3%.