Quiz 4 - Math 53
September 25, 2008

Name_____

1)[3pts] Consider the surface $-x^2 + 2x + y^2 - z^2 + 2z = 1$. Reduce the equation to one of the standard forms, classify the surface, and sketch it. In your sketch, label the point where the surface is "centered."

Completing the squares, we get $-(x-1)^2+1+y^2-(z-1)^2+1 = 1$, or $(x-1)^2-y^2+(z-1)^2 = 1$. This is a 1-sheet hyperboloid centered at $(1, 0, 1)$ "in the y direction."

2)[3pts] Let $\mathbf{r}(t)$ be a space curve for which $\mathbf{r}'''(t)$ is parallel to $\mathbf{r}(t)$. Prove that $\mathbf{r}(t) \cdot \big(\mathbf{r}'(t) \times \mathbf{r}''(t)\big)$ is constant.

By assumption, $\mathbf{r} \times \mathbf{r}''' = 0$. To show $\mathbf{r} \cdot (\mathbf{r}' \times \mathbf{r}'')$ is constant, it is enough to show it's derivative is zero.

$$\begin{aligned}\frac{d}{dt}\Big(\mathbf{r} \cdot (\mathbf{r}' \times \mathbf{r}'')\Big) &= \mathbf{r}' \cdot (\mathbf{r}' \times \mathbf{r}'') + \mathbf{r} \cdot (\mathbf{r}'' \times \mathbf{r}'' + \mathbf{r}' \times \mathbf{r}''') \\ &= (\mathbf{r}' \times \mathbf{r}') \cdot \mathbf{r}'' + 0 + \mathbf{r}' \cdot (\mathbf{r}''' \times \mathbf{r}) = 0\end{aligned}$$

3)[3pts] Consider the space curve $\mathbf{r}(t) = \langle \cos(t), \cos(2t), \sin(t) \rangle$, with $0 \le t \le 2\pi$. For what values of t is the tangent line vertical?

The direction vector of the tangent line to $\mathbf{r}(t)$ is $\mathbf{r}'(t) = \langle -\sin t, -2\sin(2t), \cos t \rangle$. For this to be vertical, we must have $-\sin t = -2\sin(2t) = 0$, which occurs only at $t = 0, \pi, 2\pi$. Since $\cos t$ is non-zero at these values, we know that $\mathbf{r}'(t)$ is actually vertical at those points.