Quiz 2 - Math 53
September 11, 2008

Name_____

Consider the polar curve given by $r = \theta^2$, $-\pi \le \theta \le \pi$.

a)[4pts] Compute $\dfrac{dy}{dx}$ as a function of θ. What is $\dfrac{dy}{dx}$ at $\theta = -\pi, -\pi/2, 0, \pi/2$, and π? Sketch this curve.

$$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{2\theta\sin\theta + \theta^2\cos\theta}{2\theta\cos\theta - \theta^2\sin\theta} = \frac{2\sin\theta + \theta\cos\theta}{2\cos\theta - \theta\sin\theta}$$

θ	$-\pi$	$-\pi/2$	0	$\pi/2$	π
dy/dx	$-\pi/2$	$4/\pi$	0	$-4/\pi$	$\pi/2$

[graph looks like a heart pointing to the left]

b)[3pts] Compute the length of the curve.

$$\text{Length} = \int_{-\pi}^{\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_{-\pi}^{\pi} |\theta|\sqrt{\theta^2+4}\, d\theta = 2\int_0^{\pi} \theta\sqrt{\theta^2+4}\, d\theta$$
$$= \int_4^{\pi^2+4} \sqrt{u}\, du = \frac{2}{3}u^{3/2}\Big|_4^{\pi^2+4} = \frac{2(\pi^2+4)^{3/2} - 16}{3} \qquad (u = \theta^2+4, du = 2\theta d\theta)$$

c)[2pts] Compute the area enclosed by the curve.

$$\text{Area} = \frac{1}{2}\int_{-\pi}^{\pi} r^2\, d\theta = \frac{1}{2}\int_{-\pi}^{\pi} (\theta^2)^2\, d\theta = \frac{1}{10}\theta^5\Big|_{\theta=-\pi}^{\pi} = \frac{\pi^5 + \pi^5}{10} = \frac{\pi^5}{5}.$$