Name_____

QUIZ 1 - MATH 53

SEPTEMBER 4, 2008

Consider the curve given by the parametric equations

$$x = 1 + \sin(t), \qquad y = \cos^2(t), \qquad -\pi/2 \le t \le \pi/2.$$

a)[3pts] Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ as functions of t.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\cos(t)\sin(t)}{\cos(t)} = -2\sin(t). \quad \frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\dfrac{dy}{dx}}{dx/dt} = \frac{-2\cos(t)}{\cos(t)} = -2.$$

b)[3pts] Set up integrals to compute the length of the curve and the area under the curve. *You do not need to evaluate the integrals.*

$$\text{Length} = \int_{-\pi/2}^{\pi/2} \sqrt{(x')^2 + (y')^2}\, dt = \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2(t) + 4\cos^2(t)\sin^2(t)}\, dt$$

$$\text{Area} = \int_{-\pi/2}^{\pi/2} y\frac{dx}{dt}\, dt = \int \cos^3(t)\, dt.$$

c)[3pts] Eliminate t and plot the curve, indicating direction with an arrow.

$(x-1)^2 + y = \sin(t)^2 + \cos(t)^2 = 1$, or $y = 1 - (x-1)^2$, with $0 \le x \le 2$, so the graph is [downward parabola through (1,1) with roots at 0 and 2]