Hanson Zhao

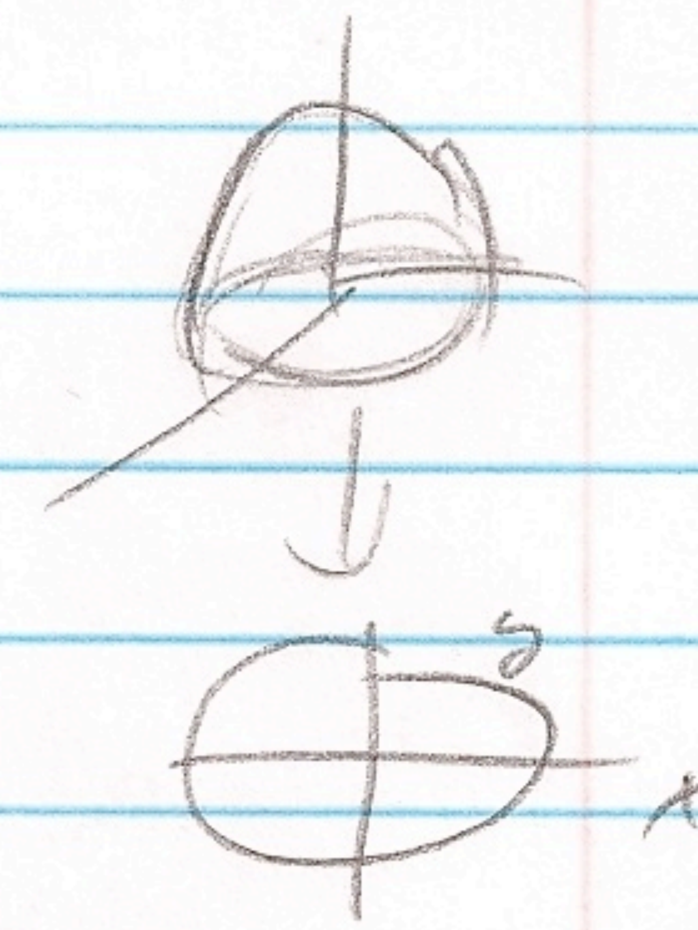
Evans 03

#7 $\int_C F \cdot dr = \iint_S \text{curl } F \cdot ds \Rightarrow$ Stokes' Thm

S is part of surface $z = 4 - (x^2 + y^2)$ $z \geq 0$

S = oriented upwards

$F = \langle x + z, x + y, x^3 \rangle$



Projection of S onto xy plane is circle $x^2 + y^2 = 4$

$0 \leq t \leq 2\pi$

$x = 2\cos t$

$y = 2\sin t$

$z = 0$

$F(r(t)) = \langle 2\cos t, 2\cos t + 2\sin t, 8\cos^3 t \rangle$

$r'(t) = \langle -2\sin t, 2\cos t, 0 \rangle$

$\int F(\vec{r}(t)) \cdot r'(t)\, dt =$

$\int_0^{2\pi} \langle 2\cos t, 2\cos t + 2\sin t, 8\cos^3 t \rangle \cdot \langle -2\sin t, 2\cos t, 0 \rangle dt =$

$\int_0^{2\pi} -4\cos t \sin t + 4\cos^2 t + 4\cos t \sin t \, dt$

$\int_0^{2\pi} 4\cos^2 t = 4\int_0^{2\pi} \frac{1}{2} - \frac{\cos 2t}{2}\, dt =$ $\quad -\frac{\sin 2\pi}{4}$

$\frac{t}{2} - \frac{\sin 2t}{4}\Big|_0^{2\pi} =$ $\boxed{\pi}$