

$$\vec{F} = \langle x\sqrt{x^2+y^2+z^2}, y\sqrt{x^2+y^2+z^2}, z\sqrt{x^2+y^2+z^2} \rangle$$

(a) To find c , we take ...

$$\text{div } \vec{F} = \sqrt{x^2+y^2+z^2} + \frac{x^2}{\sqrt{x^2+y^2+z^2}} + \sqrt{x^2+y^2+z^2} + \frac{y^2}{\sqrt{x^2+y^2+z^2}}$$

$$+ \sqrt{x^2+y^2+z^2} + \frac{z^2}{\sqrt{x^2+y^2+z^2}}$$

• we simplify to get....

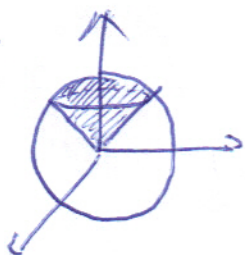
$$= 3\sqrt{x^2+y^2+z^2} + \frac{x^2+y^2+z^2}{\sqrt{x^2+y^2+z^2}}$$

• we combine the two to get...

$$= \frac{3(x^2+y^2+z^2) + (x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{4(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = 4\sqrt{x^2+y^2+z^2} \quad \text{so } \boxed{c=4}$$

(b)



$$\iint_S \vec{F} \cdot d\vec{s} = \iiint_V (\text{div } \vec{F}) dV = \iiint_{\Omega} 4\sqrt{x^2+y^2+z^2}$$

$$= 4 \int_0^{\pi/4} \int_0^{2\pi} \int_0^1 \rho^3 \sin \phi \, d\rho \, d\theta \, d\phi$$

$$= 4 (-\cos \phi) \Big|_0^{\pi/4} 2\pi \left(\frac{1}{4}\right)$$

$$\boxed{= 2\pi \left(1 - \frac{1}{\sqrt{2}}\right)}$$

Under non-acidic conditions, the double bond of a pinene reacts with reagents in a more predictable fashion. For example, epoxidation with meta-chloroperoxybenzoic acid (MCPBA) yields the epoxide, a pinene oxide, the starting material for this experiment (E1.3). Even though the epoxide is formed with no surprise, this does not mean that pinene oxide is not prone to undergoing some further chemistry.

SPRING 2007 PROBLEM 4

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④ $x^2 + y^2 + 2z^2 = 4$ ①

$z = xy$ ②

Our parametrization is:

$\left. \begin{matrix} x = x(t) \\ y = y(t) \\ z = z(t) \end{matrix} \right\}$ we plug into Eqn 1 & 2

$x(t)^2 + y(t)^2 + 2z(t)^2 = 4$

$z(t) = x(t)y(t)$

We then take derivatives w/ respect to t . using chain rule:

$(2)x(t)x'(t) + 2y(t)y'(t) + 4z(t)z'(t) = 0$ ③

$z'(t) = x'(t)y(t) + y'(t)x(t)$ ④

Since we are looking for $y'(0)$ & $z'(0)$, we rewrite eqns 3 & 4 when $t=0$

Given: $\langle x(0), y(0), z(0) \rangle = \langle 1, 1, 1 \rangle$ and $x'(0) = 1$

~~Eqn 3:~~

Eqn 3:

$2x(t)x'(t) + 2y(t)y'(t) + 4z(t)z'(t) = 0$

$= 2(1)(1) + 2(1)y'(t) + 4(1)z'(t) = 0$

$= 2 + 2y'(t) + 4z'(t) = 0$ ⑤

Eqn 4:

$z'(t) = x'(t)y(t) + y'(t)x(t)$

$z'(t) = (1)(1) + y'(t)(1)$

$z'(t) = 1 + y'(t)$ ⑥

solve for $y'(0)$ and $z'(0)$ using ⑤ & ⑥

⑤ $z'(0) = 1 + y'(0)$

⑥ $2 + 2y'(0) + 4z'(0) = 0$

$2 + 2y'(0) + 4(1 + y'(0)) = 0$

$2 + 2y'(0) + 4 + 4y'(0) = 0$

$y'(0) = -1, z'(0) = 0$

Problem 5 Spring 2007

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$$D \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle f(x, y) = \sqrt{2}x = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \cdot (f_x, f_y) \quad (1)$$

$$D \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle f(x, y) = \sqrt{2}y = \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle \cdot (f_x, f_y) \quad (2)$$

with

From (1), we get...

$$\sqrt{2}x = \frac{1}{\sqrt{2}}f_x + \frac{1}{\sqrt{2}}f_y$$

Solve for f_x & f_y

$$\sqrt{2}x + \sqrt{2}y = \frac{1}{\sqrt{2}}f_x + \frac{1}{\sqrt{2}}f_x + \frac{1}{\sqrt{2}}f_y - \frac{1}{\sqrt{2}}f_y$$

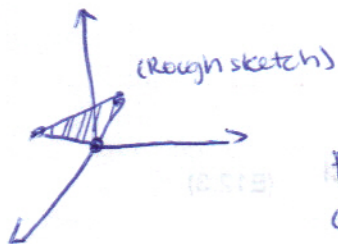
$$\sqrt{2}(x+y) = \frac{2}{\sqrt{2}}f_x$$

$$2(x+y) = 2f_x$$

$$\boxed{x+y = f_x}$$

$$\boxed{x-y = f_y}$$

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Point A = (0,0,0)
 Point B = (1,0,1)
 Point C = (1,1,2)

First, we find 2 vectors so that we can find an eqn for the plane of the triangle.

$$\vec{BA} = \langle 1-0, 0-0, 1-0 \rangle = \langle 1, 0, 1 \rangle$$

$$\vec{CA} = \langle 1-0, 1-0, 2-0 \rangle = \langle 1, 1, 2 \rangle$$

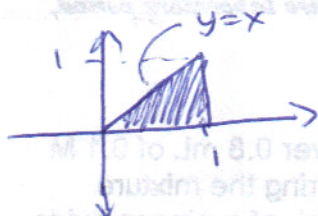
$$\langle 1, 0, 1 \rangle \times \langle 1, 1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix} = (-1)\mathbf{i} - 1\mathbf{j} + 1\mathbf{k}$$

Eqn to plane:

$$(-1)(x-0) + (-1)(y-0) + (1)(z-0) = 0$$

$$-x - y + z = 0 \Rightarrow \underline{z = x + y}$$

The shadow of the triangle in the x-y plane is:



$$\begin{aligned} \iint_S \langle 3, 4, 5 \rangle \cdot d\mathbf{S} &= \int_0^1 \int_0^x (-3-4+5) dA \\ &= -2 \int_0^1 \int_0^x dy dx \\ &= -2 \int_0^1 x dx \\ &= \left(-\frac{1}{2} \right) x^2 \Big|_0^1 = \boxed{-1} \end{aligned}$$