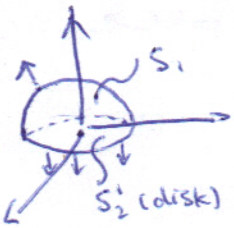
Spring 2007 Problem 9

Jonathan Wang

a.)



$F = \langle x + y^2 + z^2, x^2 - y + z^2, x^2 + y^2 \rangle$

Divergence Thm:

$\iint_S \vec{F} \cdot d\vec{s} = \iiint_E \operatorname{div} \vec{F} \, dV = \iiint_E (1 - 1 + 0) \, dV$

$= \iiint_E 0 \, dV = 0$

same

$\iint_{S_1} \vec{F} \cdot d\vec{s} + \iint_{S_2'} \vec{F} \cdot d\vec{s}$

so, that means...

$\boxed{\iint_{S_1} \vec{F} \cdot ds = -\iint_{S_2'} F \cdot ds = \iint_{S_2} \vec{F} \cdot d\vec{s}}$

so S_2 is the disk with upward orientation

b.)

$\iint_{S_1} \vec{F} \cdot d\vec{s} = \iint_{S_2} \vec{F} \cdot d\vec{s}$

$= \iint_{S_2} (x^2 + y^2) \, dA = \int_0^{2\pi} \int_0^1 r^3 \, dr \, d\theta$

$= 2\pi \left(\frac{1}{4}\right) = \boxed{\frac{\pi}{2}}$