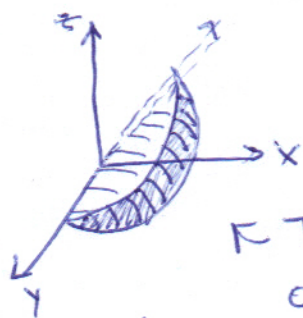


2 SPRING 07 PROBLEM 2

Jonathan Wong

Bounded by:  $x^2 + y^2 + z^2 = 4$

$z = x$



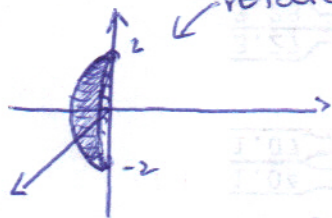
This shape is like a wedge, being  $\frac{1}{8}$  of the volume of a sphere so:

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$r = 2$

$$\underbrace{\left(\frac{1}{8}\right)}_{\text{fraction of sphere observed}} \underbrace{\frac{4}{3} \pi (2)^3}_{\text{volume of sphere}} = \boxed{\frac{4}{3} \pi}$$

or:



rotated from since it is symmetrical.

$$0 \leq \rho \leq 2$$

$$0 \leq \phi \leq \pi$$

$$0 \leq \theta \leq \pi/4$$

$$\int_0^\pi \int_0^{\pi/4} \int_0^2 1 \, dV$$

we change to spherical coords.

$$\int_0^\pi \int_0^{\pi/4} \int_0^2 \underbrace{\rho^2 \sin \phi}_{\text{magnification factor}} \, d\rho \, d\theta \, d\phi = \boxed{\frac{4}{3} \pi}$$