DAVIS C1

Hutchings spring 07. final

10. Let C be semicircle $x^2+y^2=1$, $y \geq 0$ oriented counterclockwise.

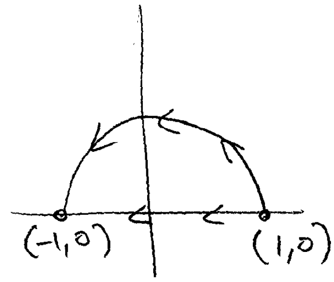
Calculate the line integral

$$\int_C (-y+\cos x)\,dx + (x+\sin y)\,dy$$

$$\int_C F \cdot d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$$

$$\vec{F} = \langle -y+\cos x, x+\sin y \rangle$$

$$\vec{F} = \underbrace{\langle \cos x, \sin y \rangle}_{\vec{F}=\nabla f,\ \boxed{f = \sin x - \cos y}} + \langle -y, x \rangle$$



So $\int_C (-y+\cos x)\,dx + (x+\sin y)\,dy$

$$= \int_C \cos x\,dx + \sin y\,dy + \int_C -y\,dx + x\,dy$$

$= \int_C \nabla f \cdot d\vec{r}$

and by fund. thm. of line integrals

$$\int_C \nabla f \cdot d\vec{r} = f(x_2,y_2) - f(x_1,y_1)$$

$$= f(-1,0) - f(1,0)$$

$$= [\sin(-1) - 1] - [\sin 1 - 1]$$

$$= -2\sin(1)$$

$x = \cos t$ $\quad dx = -\sin t\,dt$

$y = \sin t$ $\quad dy = \cos t\,dt$

$$\int_C [-\sin t(-\sin t) + \cos t(\cos t)]\,dt$$

$$= \int_0^\pi \sin^2 t + \cos^2 t\,dt$$

$$= \int_0^\pi 1\,dt$$

$$= \pi$$

So $\int_C F \cdot d\vec{r} = \boxed{-2\sin(1) + \pi}$