Salar Assadian

Final: 12/15/07

1. Consider space curve $r(t) = \langle 1, t, t^2 \rangle$. Let f be a function on R^3 satisfying $\nabla f(1,2,4) = \langle 5,6,7 \rangle$. Calculate $\frac{d}{dt} f(r(t))\Big|_{t=2}$

$$\frac{d}{dt} f(r(t)) = f'(r(t)) \cdot r'(t)$$

$$\left.\begin{aligned} r(2) &= \langle 1,2,4 \rangle \\ \nabla f(r(2)) &= \langle 5,6,7 \rangle \end{aligned}\right\} \text{Given in problem}$$

$$r'(t) = \langle 0, 1, 2t \rangle$$

$$r'(2) = \langle 0, 1, 4 \rangle$$

$$f'(r(2)) \cdot r'(2)$$

$$\langle 5,6,7 \rangle \cdot \langle 0,1,4 \rangle$$

$$0 + 6 + 28$$

$$\boxed{34}$$

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2. Find the volume of solid region consisting of all points that are above the surface $z=x^2$ and below surface $z=4-y^2$.

$$\iiint_R 1\, dV$$

$$\iiint 1\, dz\, dy\, dx \quad ; \qquad x^2 < z < 4-y^2$$

But we covert to cylindrical polar or else it is too hard to solve

$$\int_0^{2\pi}\int_0^{2}\int_{r^2\cos^2\theta}^{4-r^2\sin^2\theta} r\, dz\, dr\, d\theta \qquad \begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r^2\cos^2\theta &< z < 4-r^2\sin^2\theta \end{aligned}$$

$$\int_0^{2\pi}\int_0^{2} (4-r^2\sin^2\theta - r^2\cos^2\theta)r\, dr\, d\theta$$

to determine r, make $z=\phi$ to "determine" a disk

$$x^2 = 4-y^2$$

$$x^2+y^2 \leq 4$$

$$0 \leq r \leq 2$$

$$\int_0^{2\pi}\int_0^{2} (4-r^2)r\, dr\, d\theta$$

$$\int_0^{2\pi} 2r^2 - \frac{r^4}{4}\Big|_0^2\, d\theta$$

$$\int_0^{2\pi} (8-4)\, d\theta$$

$$4 * 2\pi$$

$$8\pi$$

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4. Calculate double integral $\iint_D xy\,dA$ where D is the region in the plane defined by:

$$x^4+y^4 \leq 1 \quad x \geq 0 \quad y \geq 0$$

$u = x^2 \quad v = y^2$

$\sqrt{u} = x \quad \sqrt{v} = y$

$u^2+y^2 \leq 1 \quad u \geq 0 \quad v \geq 0$

Jacobian:

$$\begin{vmatrix} 0.5u^{-1/2} & 0 \\ 0 & 0.5v^{-1/2} \end{vmatrix} = \frac{1}{4} \cdot \frac{1}{\sqrt{uv}}$$

$$\iint f(u,v) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$\iint_{D'} \cancel{\sqrt{uv}} \cdot \frac{1}{4\cancel{\sqrt{uv}}}\, du\,dv$$

$$\frac{1}{4}\iint_{D'} 1 \cdot du\,dv$$

convert to polar to make life easier

$$\frac{1}{4}\int_0^{2\pi}\int_0^1 r \cdot dr\,d\theta \Rightarrow \frac{1}{4} \cdot 2\pi \int_0^1 r\,dr \Rightarrow \frac{1}{4} \cdot 2\pi \cdot \frac{1}{2}$$

$u^2+v^2 \leq 1$

$0 \leq r \leq 1$

$\pi/4$

Because we look at 1/4 of circle

$\frac{1}{4} \cdot \pi/4 = \boxed{\pi/16}$

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6. Find the tengent plane to parametrized surface:

$$r(u,v) = \langle v-1,\ uv,\ u^3+2 \rangle$$

at point $(1,2,3)$.

- Determine r_u and r_v:

$$r_u = \langle 0, v, 3u^2 \rangle$$

$$r_v = \langle 1, u, 0 \rangle$$

- $r_u \times r_v$ to get normal vector:

$$\begin{array}{ccc} i & j & k \\ 0 & v & 3u^2 \\ 1 & u & 0 \end{array} \Rightarrow \langle 0-3u^3,\ 3u^2-0,\ 0-v \rangle \Rightarrow \langle -3u^2, 3u^2, -v \rangle$$

Determine cross product "values" at Point $(1,2,3)$:

$x=1$	$y=2$	$z=3$
$v-1=1$	$uv=2$	$u^3+2=3$
$2-1=1$	$v=\frac{2}{1}$	$u=1$ ✓
$1=1$ ✓	$v=2$ ✓	

Now we know u and v, plug into cross product to find normal vector:

$$\langle -3(1)^2, 3(1)^2, -2 \rangle$$

$$\langle -3, 3, -2 \rangle$$

Now dot product normal vector with coordinates:

$$\langle -3, 3, -2 \rangle \cdot \langle x-1, y-2, z-3 \rangle = -3(x-1)+3(y-2)-2(z-3) = 0 \rightarrow -3x+3y-2z=-3$$