Hutchings fall 07 final

DAVIS C1

9. calculate $\int_C \vec{F} \cdot d\vec{r}$,

C is triangle $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, oriented counterclockwise when viewed from above

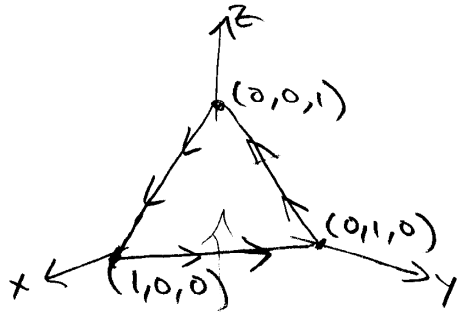
and $\vec{F} = \langle \sin^2 x + y, \sin^2 y + z, xy \rangle$

Stokes theorem:

$$\int_C \vec{F} \cdot d\vec{r} = \iint \text{curl } F \cdot d\vec{S}$$

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \sin^2 x + y, & \sin^2 y + z, & xy \end{vmatrix}$$

$$= (x-1)\hat{i} - y\hat{j} - \hat{k}$$



$z = 1 - x - y$

$\vec{r} = \langle x, y, 1-x-y \rangle$

$\vec{r}_x = \langle 1, 0, -1 \rangle$

$\vec{r}_y = \langle 0, 1, -1 \rangle$

$(r_x \times r_y) = \hat{i} + \hat{j} + \hat{k}$
$= \langle 1, 1, 1 \rangle$

$$\iint \langle x-1, -y, -1 \rangle \cdot \langle 1, 1, 1 \rangle \, dA$$

$$= \iint x - 1 - y - 1 \, dA$$

$$= \int_0^1 \int_0^{1-x} x - y - 2 \, dy \, dx$$

$$= \int_0^1 xy - \frac{y^2}{2} - 2y \Big|_0^{1-x} dx$$

$$= \int_0^1 (x - x^2) - \frac{(1-x)^2}{2} - 2(1-x) \, dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{(1-x)^3}{3} + (1-x)^2 \Big|_0^1 = \left[\frac{1}{2} - \frac{1}{3}\right] - \left[\frac{1}{3} + 1\right]$$

$$= \frac{1}{2} + \frac{2}{3} + 1 = \boxed{\frac{13}{6}}$$