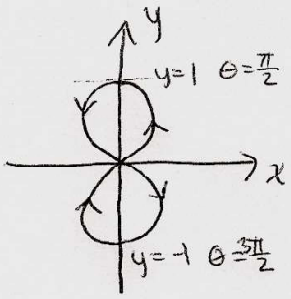
⑧ a) sketch the parametric curve

$x = \sin\theta\cos\theta$, $y = \sin\theta$ $\quad 0 \le \theta \le 2\pi$



b) Calculate Area Enclosed

$\text{Area} = \int_c x\,dy$

going from $0 \le \theta \le 2\pi$

$\text{Area} = 4\int_c x\,dy$ where $0 \le \theta \le \frac{\pi}{2}$

$= 4\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \cdot \cos\theta\,d\theta$

$= 4\int_0^{\frac{\pi}{2}} \sin\theta\cos^2\theta\,d\theta$

$u = \cos\theta \quad du = -\sin\theta$

u goes from 1 to 0

$4\int_1^0 -u^2\,du = -\frac{4}{3}u^3\Big|_1^0$

$= \boxed{\frac{4}{3}}$

To Draw

① $0 \le \theta \le \frac{\pi}{2}$

as θ goes to $\frac{\pi}{2}$ what happens to x and y?

x: increases, then decrease, ends at 0

y: goes to 1

② $\frac{\pi}{2} \le \theta \le \pi$

x: $\sin\theta \to$ positive, $\cos\theta \to$ negative

so $x \to$ negative

ends at 0 because $\sin\pi = 0$

y: goes to 0

③ $\pi \le \theta \le \frac{3\pi}{2}$

x: $\sin\theta \to$ negative, $\cos\theta \to$ negative

so $x \to$ positive.

increase, decreases, ends at 0

y: $\sin\theta$ is negative so y is negative

ends at -1.

④ $\frac{3\pi}{2} \le \theta \le 2\pi$

x: $\sin\theta \to$ negative, $\cos\theta \to$ positive

so x = negative

ends at 0 because $\sin 2\pi = 0$

y: $\sin\theta$ negative so y = negative

goes from -1 to 0.