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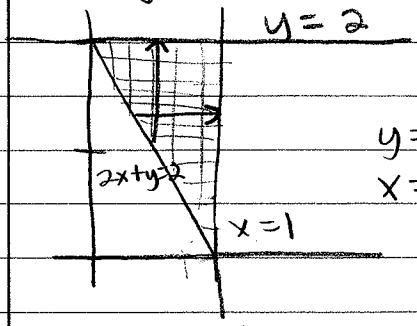
Hutchings Midterm #2

① (a) Let D be the triangle bounded by the lines $x=1$, $y=2$, and $2x+y=2$. Give two expressions for the double integral $\iint_D e^{x-y}\,dA$ as an iterated integral.



$$\int_0^2 \int_{1-y/2}^{1} e^{x-y}\,dx\,dy$$

$y = 2-2x$

$x = \frac{2-y}{2}$

$$\int_0^1 \int_{2-2x}^{2} e^{x-y}\,dy\,dx$$

(b) Evaluate the above double integral.

$$\int_0^1 \int_{2-2x}^{2} e^{x-y}\,dy\,dx$$

$$= \int_0^1 -e^{x-y}\Big|_{y=2-2x}^{y=2}\,dx$$

$$= \int_0^1 -\left[e^{x-2} - e^{3x-2}\right]dx$$

$$= \int_0^1 -e^{x-2}\,dx + \int_0^1 e^{3x-2}\,dx$$

$$= -e^{x-2}\Big|_0^1 + \tfrac{1}{3}e^{3x-2}\Big|_0^1$$

$$= \left[-e^{-1} - (-e^{-2})\right] + \tfrac{1}{3}\left[e - e^{-2}\right]$$

$$= \left[-\frac{1}{e} + \frac{1}{e^2}\right] + \tfrac{1}{3}\left[e - \frac{1}{e^2}\right]$$

$$= \left[\frac{1-e}{e^2}\right] + \left[\frac{e^3-1}{3e^2}\right]$$

$$= \left[\frac{3-3e}{3e^2}\right] + \left[\frac{e^3-1}{3e^2}\right]$$

$$= \boxed{\frac{2-3e+e^3}{3e^2}}$$