Fall 1996 2ND Midterm

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1. (a) Use Lagrange Multipliers & let $f(x,y) = (x+1)^2 + y^2$ $\quad g(x,y) = x^2 + \frac{y^2}{4} = 1$

$\nabla f = \lambda \nabla g$

$\langle 2x+2, 2y \rangle = \lambda \langle 2x, \frac{y}{2} \rangle$

$2x+2 = \lambda 2x \Rightarrow \lambda = \frac{2x+2}{2x} = \frac{x+1}{x}$ if $x \neq 0$

$2y = \lambda \frac{y}{2} \Rightarrow \lambda = (2y)\left(\frac{2}{y}\right) = 4$ if $y \neq 0$

$\frac{x+1}{x} = \lambda = 4$

$\frac{x+1}{x} = 4$

$x+1 = 4x$

$1 = 3x$

$x = \frac{1}{3}$

constraint: $x^2 + \frac{y^2}{4} = 1$

$\left(\frac{1}{3}\right)^2 + \frac{y^2}{4} = 1$

$\frac{y^2}{4} = \frac{8}{9}$

$y = \pm\sqrt{\frac{32}{9}}$

$\boxed{\left(\frac{1}{3}, \sqrt{\frac{32}{9}}\right)}$ $\quad \boxed{\left(\frac{1}{3}, -\sqrt{\frac{32}{9}}\right)}$

$\boxed{f\left(\frac{1}{3}, \sqrt{\frac{32}{9}}\right) = \frac{48}{9}}$ $\quad \boxed{f\left(\frac{1}{3}, -\sqrt{\frac{32}{9}}\right) = \frac{48}{9}}$

max $\quad$ max

Also consider situation if $y=0$

constraint $x^2 + \frac{y^2}{4} = 1$

$x^2 = 1$

$x = \pm 1$

$(1,0)$ $\quad \boxed{(-1,0)}$

$f(1,0) = 4$ $\quad \boxed{f(-1,0) = 0}$

min

Also consider situation if $x=0$

constraint $x^2 + \frac{y^2}{4} = 1$

$\frac{y^2}{4} = 1 \Rightarrow y = \pm 2$

$(0,2)$ $\quad (0,-2)$

$f(0,2) = 5$ $\quad f(0,-2) = 5$

(b) let $f = (x+1)^2 + y^2$ $\quad$ let $g = \left(x^2 + \frac{y^2}{4}\right)$

$\nabla f = \langle 2x+2, 2y \rangle$ $\quad \nabla g = \langle 2x, \frac{y}{2} \rangle$

$\nabla f(-1,0) = \langle 0,0 \rangle$ $\quad \nabla g(-1,0) = \langle -2, 0 \rangle$

$\nabla f\left(\frac{1}{3}, \sqrt{\frac{32}{9}}\right) = \langle 2\frac{2}{3}, 2\sqrt{\frac{32}{9}} \rangle$ $\quad \nabla g\left(\frac{1}{3}, \sqrt{\frac{32}{9}}\right) = \langle \frac{2}{3}, \frac{1}{2}\sqrt{\frac{32}{9}} \rangle$

$\nabla f\left(\frac{1}{3}, -\sqrt{\frac{32}{9}}\right) = \langle 2\frac{2}{3}, -2\sqrt{\frac{32}{9}} \rangle$ $\quad \nabla g\left(\frac{1}{3}, -\sqrt{\frac{32}{9}}\right) = \langle \frac{2}{3}, -\frac{1}{2}\sqrt{\frac{32}{9}} \rangle$