Monica Chan

F96 Final Exam - Bergman

⑤ $\vec{F}_2 = \langle 3xz, -5yz, z^2 \rangle$

ⓐ Find const p + q so $\text{curl}(pyz^2\hat{i} + qxyz\hat{k}) = \vec{F}_2$

$$\text{curl}(\langle pyz^2, 0, qxyz \rangle) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ pyz^2 & 0 & qxyz \end{vmatrix} = \langle qxz, 2pyz - qyz, -pz^2 \rangle$$

$qxz = 3xz$

$\boxed{q = 3}$

$2pyz - 3yz = -5yz$

$2pyz = -2yz$

$\boxed{p = -1}$

$z^2 = -pz^2$

$p = -1$

ⓑ Calc $\iint_S \vec{F} \cdot d\vec{S}$, S is part of $z = e^{x^2+y^2}$ below plane $z = e$, w/ upward orientation

(Use part (a))

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\,\vec{F} \cdot d\vec{S} = \int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$\vec{F}_1 = \langle -yz^2, 0, 3xyz \rangle$

$$= \int_0^{2\pi} (e^2 \sin^2 t)\,dt$$

$\vec{r}(t) = \langle \cos t, \sin t, e \rangle$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, 0 \rangle$

$e = e^{x^2+y^2}$

$\therefore x^2 + y^2 = 1$

$$= e^2 \int_0^{2\pi} \left(\frac{1 - \cos 2t}{2} \right) dt$$

$$= e^2 \left(\frac{1}{2}t - \frac{\sin 2t}{4} \right) \Big|_0^{2\pi}$$

$\vec{F}(\vec{r}(t)) = \langle -e^2 \sin t, 0, 3e \sin t \cos t \rangle$

$$= \boxed{\pi e^2}$$

$0 \le t \le 2\pi$