Derrick Cheng
Midterm #2
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C is a plane curve at (0,0) and ending at (1,1)

$F = \langle x^2+y, y^2+x \rangle$

5.(a) $\int_C F \cdot dr$ has the same value for "every C as above" if F is conservative

F is conservative because

① $F = \langle x^2+y, y^2+x \rangle$ on an open simply-connected region D

② $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ $F = Pi + Qj$

$P = x^2+y$ $Q = y^2+x$

$\frac{\partial P}{\partial y} = 1$ $\frac{\partial Q}{\partial x} = 1$

$\Rightarrow \frac{\partial P}{\partial y} = 1 = \frac{\partial Q}{\partial x}$

Since ① and ② are satisfied, F is conservative by Thereom 6 (Pg. 746)

Since F is conservative, we can apply the fundamental thereom of ~~[illegible]~~ line integrals $\int_C \nabla f \cdot dr = f(r(b)) - f(r(a))$

$\Rightarrow$ The values of $\int_C F \cdot dr$ only depends on the end points, not on the path itself

$\Rightarrow$ Since the start and endpoints of curve C is always (0,0) & (1,1), $\int_C F \cdot dr$ has the same value for every C as above

QED ✓

(b)

$\int_C F \cdot dr = f(r(b)) - f(r(a))$

$= f(\langle 1,1 \rangle) - f(\langle 0,0 \rangle)$

$= 1\frac{2}{3} - 0$

$= \boxed{1\frac{2}{3}}$

$\int P\,dx = \frac{x^3}{3} + xy$

$\int Q\,dy = \frac{y^3}{3} + xy$

$f(x,y) = \frac{x^3}{3} + xy + \frac{y^3}{3} + k$

$f\langle 1,1 \rangle = \frac{1}{3} + 1 + \frac{1}{3} = 1\frac{2}{3}$

$f\langle 0,0 \rangle = 0$