Math53 MT 1 10/2/03 Hutching

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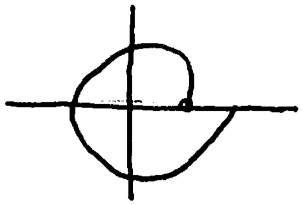
1) a) Sketch the curve $r = e^{2\theta}$ $\quad 0 \le \theta \le 2\pi$



b) Find length

$$L = \int_{\theta_0}^{\theta_1} \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{e^{4\theta} + 4e^{4\theta}} = \sqrt{5}\int_0^{2\pi} e^{2\theta}$$

$$= \frac{\sqrt{5}}{2}\left(e^{2\theta}\right)\Big|_0^{2\pi} = \frac{\sqrt{5}}{2}\left(e^{4\pi} - 1\right)$$

2.) Show that the limit $\lim\limits_{(x,y)\to(1,1)} \frac{x+2y-3}{x+y-2}$ does not exist

$x=1 \quad \lim\limits_{y\to 1} \frac{2(y-1)}{(y-1)} = 2$

$y=1 \quad \lim\limits_{x\to 1} \frac{x-1}{x-1} = 1$

So lim DNE

3.) $\frac{\partial f}{\partial x} = \frac{1}{x}$, $\frac{\partial f}{\partial y} = \frac{1}{y}$ $\quad x = r\cos\theta \quad y = r\sin\theta$

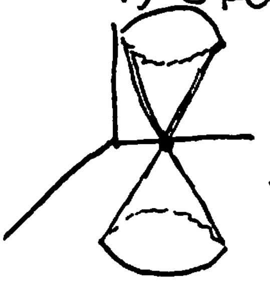
calculate $\frac{\partial f}{\partial r}$, $\frac{\partial f}{\partial \theta}$ in terms of r and θ

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} = \left(\frac{1}{r\cos\theta}\right)\cos\theta + \left(\frac{1}{r\sin\theta}\right)\sin\theta = \boxed{\frac{2}{r}}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} = \left(\frac{1}{r\cos\theta}\right)(-r\sin\theta) + \left(\frac{1}{r\sin\theta}\right)(r\cos\theta)$$

$$= -\frac{\sin}{\cos} + \frac{\cos}{\sin} = \boxed{\cot\theta - \tan\theta}$$

4) a) Sketch surface $x^2+(y-1)^2=z^2$



b) find tangent plane @ $(4,4,5)$

$x^2+(y-1)^2-z^2=0=f(x,y,z)$

$f_x(4,4,5)=8$ $\quad f_y(4,9,5)=6$ $\quad f_z(4,4,5)=-10$

$8(x-4)+6(y-4)-10(z-5)=0$ $\quad \boxed{8x+6y-10z=6}$

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normal vectors to the planes $2x+2y+z=5$, $2x-y-2z-1$

5.) $n_1=\langle 2,2,1\rangle$ $\quad n_2=\langle 2,-1,-2\rangle$

a) $n_1 \times n_2$ = normal vector to plane

$\langle 2,2,1\rangle \times \langle 2,-1,-2\rangle = \langle -3,6,-6\rangle = \langle -1,2,-2\rangle$

b.) The planes intersect on line L. What $\measuredangle$ do the 2 planes intersect

$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{4-2-2}{5} = 0$ $\quad \boxed{\theta=\pi/2}$

c.) Find a tangent vector to the line L.

intersection $\quad y+z=2 \quad r=r_0+tv$

$2y+z=5 \quad y+2z=1$

$z=5-2y \quad y+10-4y=1$ $\quad \boxed{r=\langle 0,3,-1\rangle+\langle -1,2,-2\rangle+}$

$-3y=-9$

$y=3 \quad z=-1 \quad x=0$

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6) $\quad 2e^{x+2y+3z}=4 \quad z=\frac{4}{e^{x+2y+3z}}$

$\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}\left(4\left(e^{-x-2y-3z}\right)\right) = -4e^{-x-2y-3z}\left(-3\frac{\partial z}{\partial x}-1\right)$

$\frac{\partial z}{\partial x}\left(1+\frac{12}{e^{x+2y+3z}}\right) = \frac{-4}{e^{x+2y+3z}} = \frac{\partial z}{\partial x}\left(\frac{e^{x+2y+3z}+12}{e^{x+2y+3z}}\right)$

$\boxed{\frac{\partial z}{\partial x} = \frac{-4}{(e^{x+2y+3z}+12)}}$

$\partial z/\partial y = -8/(e^{x+2y+3z}+12)$