VINOD CHANDRU
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Sec. 111

Fall 03 Final Exam - Hutchings

9. Either find, or prove that there doesn't exist:

a) a function f on $\mathbb{R}^3$ such that $\nabla f = \langle y, x+z\cos y, \sin y\rangle$

b) a vector field F on $\mathbb{R}^3$ such that $\nabla \times F = \langle z, y, x\rangle$.

Solution: a) $\nabla f = \langle y, x+z\cos y, \sin y\rangle$.

$$f_x(x,y,z) = y,\ f_y(x,y,z) = x+z\cos y,\ f_z(x,y,z) = \sin y$$

$$f(x,y,z) = xy + g(y,z)$$

$$\therefore f_y(x,y,z) = x + g_y(y,z) = x + z\cos y$$

$$\therefore g(y,z) = z\sin y + h(z)$$

$$\Rightarrow f(x,y,z) = xy + z\sin y + h(z)$$

$$\therefore f_z(x,y,z) = \sin y + h_z(z) = \sin y$$

$$\therefore h(z) = K$$

$$\boxed{f(x,y,z) = xy + z\sin y + K.}$$

b) If F exists, div curl F should be 0.

$$\text{curl } F = \nabla \times F = \langle z, y, x\rangle$$

$$\therefore \text{div curl } F = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(x) = 0+1+0 = \underline{\underline{1}} \neq 0$$

$\boxed{\therefore F \text{ does not exist.}}$