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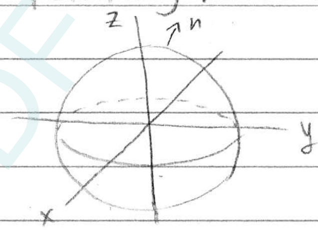
Fall 2003 Hutchings

8. Calculate $\iint_S F \cdot dS$, where S is the unit sphere $x^2+y^2+z^2=1$, oriented using the outward pointing normal, and

$$F = \langle x+\sin y,\ y+\sin z,\ z+\sin x \rangle$$



$$\iint_S F \cdot dS = \iiint_E \operatorname{div} F \, dV$$ (divergence theorem)

$$\operatorname{div} F = \frac{\partial}{\partial x}(x+\sin y) + \frac{\partial}{\partial y}(y+\sin z) + \frac{\partial}{\partial z}(z+\sin x)$$

$$= 1 + 1 + 1 = 3$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi \, d\phi \int_0^1 3\rho^2 \, d\rho$$

$$= \theta\Big]_0^{2\pi} \cdot -\cos\phi\Big]_0^{\pi} \cdot \frac{3\rho^3}{3}\Big]_0^1$$

$$= [2\pi - 0] \cdot [-\cos\pi + \cos 0] \cdot [(1)^3 - (0)^3]$$

$$= [2\pi] \cdot [1+1] \cdot [1] = 2\pi \cdot 2 \cdot 1 = \boxed{4\pi}$$