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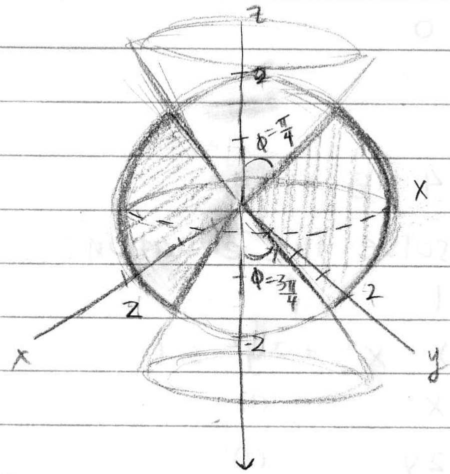
math 53 Final 12/18/03 Hutchings

7. Calculate the volume of a region consisting of all points that are inside the sphere $x^2+y^2+z^2=4$, below the cone $z=\sqrt{x^2+y^2}$, and above the cone $z=-\sqrt{x^2+y^2}$



$x=\rho\sin\phi\cos\theta \quad y=\rho\sin\phi\sin\theta \quad z=\rho\cos\phi$

$\rho^2=x^2+y^2+z^2 \quad r=\rho\sin\phi$

· The equation of the upper cone can be written as:

$\rho\cos\phi=\sqrt{\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta}$

$=\rho\sin\phi$

$\cos\phi=\sin\phi$

$\phi=\pi/4. \quad \pi/4\le\phi\le\frac{3\pi}{4}$

$0\le\theta\le 2\pi \quad , \quad 0\le\rho\le 2$

$$\int_{\pi/4}^{3\pi/4}\int_0^{2\pi}\int_0^2 \rho^2\sin\phi\, d\rho\, d\theta\, d\phi$$

$$\int_{\pi/4}^{3\pi/4}\sin\phi\, d\phi\int_0^{2\pi}d\theta\int_0^2\rho^2\, d\rho$$

$$=\left[\left[-\cos\phi\right]_{\frac{\pi}{4}}^{3\pi/4}\right]\left[\left[\theta\right]_0^{2\pi}\right]\left[\left[\frac{\rho^3}{3}\right]_0^2\right]$$

$$=\left[-\cos\frac{3\pi}{4}+\cos\frac{\pi}{4}\right]\left[2\pi-0\right]\left[\frac{8}{3}-0\right]=\left[\frac{\sqrt2}{2}+\frac{\sqrt2}{2}\right]\left[2\pi\right]\left[\frac{8}{3}\right]$$

$$=\left[\frac{\cancel{2}\sqrt2}{\cancel{2}}\right]\left[2\pi\right]\left[\frac{8}{3}\right]=\frac{16\pi\sqrt2}{3}$$