Math 53 Final, 12/18/03 Hutchings

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Sec 111

(3) Find the absolute minimum and maximum values of $f = x^2 - 4x + y^2$ subject to the constraint $x^2 + 2y^2 \le 1$.

$f = x^2 - 4x + y^2$

$f_x = 2x - 4 = 0$ $\qquad$ $f_y = 2y = 0$

$2x = 4$ $\qquad$ $y = 0$

$x = 2$

$(2)^2 + (2 \cdot 0)^2 \le 1$

$4 \le 1$ <u>FALSE!</u> $\therefore$ $(2,0)$ is not a critical pt.

$\nabla f = \lambda \nabla g$, $f = x^2 - 4x + y^2$ $\qquad$ $g = x^2 + 2y^2 = 1$

$\langle 2x - 4, 2y \rangle = \lambda \langle 2x, 4y \rangle$

$2x - 4 = \lambda 2x$ $\qquad$ $2y = \lambda 4y$

$2(x-2) = 2\lambda x$ $\qquad$ $y = \lambda 2y \longrightarrow y = 0$

$x - 2 = \lambda x$ $\qquad$ $\downarrow$ $\qquad$ $x^2 + (2 \cdot 0)^2 = 1$

$x - 2 = \frac{1}{2}x \longleftarrow \lambda = \frac{1}{2}$ $\qquad$ $x^2 = 1$

$\frac{1}{2}x = 2$ $\qquad$ $x = \pm 1$

$x = 4$ $\qquad$ pts: $(1,0)$, $(-1,0)$

$(4)^2 + 2y^2 = 1$

$2y^2 = -15 \longrightarrow$ <u>NOT POSSIBLE!</u>

$\therefore \lambda \ne \frac{1}{2}$

$f(1,0) = (1)^2 - 4(1) + (0)^2$
$= 1 - 4 + 0 = -3$

$f(-1,0) = (-1)^2 - 4(-1) + (0)^2$
$= 1 + 4 + 0 = 5$

Absolute Maximum: $f(-1,0) = 5$
Absolute Minimum: $f(1,0) = -3$