

5th November

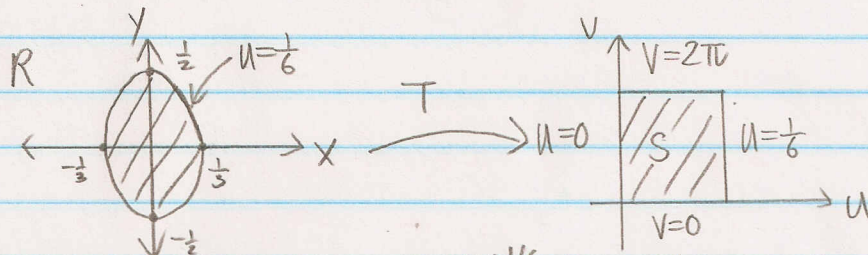
Fall 2003 Hutchings: Final Exam

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10. Let R be the region $9x^2 + 4y^2 \leq 1$ in the xy plane. Calculate $\iint_R (9x^2 + 4y^2)^{5/2} dA$

$$\text{let } \begin{cases} x = 2u \cos v \\ y = 3u \sin v \end{cases} \quad \begin{cases} 9x^2 + 4y^2 = 36u^2(\cos^2 v + \sin^2 v) \\ = 36u^2 \end{cases}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2\cos v & -2u\sin v \\ 3\sin v & 3u\cos v \end{vmatrix} = 6u\cos^2 v + 6u\sin^2 v = 6u$$



$$\begin{aligned} \iint_R (9x^2 + 4y^2)^{5/2} dA &= \int_0^{2\pi} \int_0^{1/6} (36u^2)^{5/2} 6u du dv \\ &= \int_0^{2\pi} \int_0^{1/6} 6^6 u^6 du dv \\ &= \int_0^{2\pi} 6^6 \left(\frac{1}{7}u^7\right) \Big|_0^{1/6} dv \\ &= \frac{1}{42} (2\pi) = \pi/21 \end{aligned}$$